



**Jueves, 1 de junio de 2017.**

**César Palencia (Universidad de Valladolid)**

## Some contributions to Crouzeix's Conjecture

### Abstract:

In the first part of the talk we will give examples of evolutionary equations of interest (PDE and integro-differential), not necessarily linear ones, for which the study of the well posedness of the equations themselves, the analysis of their discretizations and the numerical linear algebra involved bring about the need of estimating the operator norm  $\|f(\mathbf{A})\|$  of suitable holomorphic mappings  $f$  evaluated at linear operators  $\mathbf{A}$  on a Hilbert or Banach space, which is the central issue of the present conference. Then, in the context of complex Hilbert spaces  $(H, \langle \cdot, \cdot \rangle)$ , we will review the dilation theory and some classic, relevant results for the unit disk due to J. Von Newman, Berger, Stampfli and Ando, showing some applications. At this point we will focus on a theorem due to the brothers B. and F. Delyon: given an open and convex subset  $\Omega \subset \mathbf{C}$ , there exists  $C_\Omega > 0$  such that the bound

$$\|f(\mathbf{A})\| \leq C_\Omega \|f\|_\infty = C_\Omega \sup_{z \in \Omega} |f(z)|$$

is valid whenever  $\mathbf{A}$  is a linear operator on  $H$  whose numerical range  $W(\mathbf{A}) := \{ \langle \mathbf{x}, \mathbf{x} \rangle : \mathbf{x} \in H, \|\mathbf{x}\| = 1 \}$  is contained in  $\Omega$  and  $f : \Omega \subset \mathbf{C} \rightarrow \mathbf{C}$  is holomorphic. Later M. Crouzeix showed that, for all  $\Omega \subset \mathbf{C}$  as mentioned,  $C_\Omega < 11.08$  and conjectured that  $C_\Omega \leq 2$  (an estimate known to be valid in case  $\Omega$  is a disk), a conjecture that has arisen a lot of interest and, although there is numerical support to it, remains an open problem. After commenting on some implications of the universality of  $C_\Omega$ , the final part of the talk, more technical, we will devoted to a new approach which is based on potential theory. In this way it becomes clear that  $C_\Omega \leq 3$  and, after some considerations, we will also present the recent improvement  $C_\Omega \leq 1 + \sqrt{2}$ .



Univ. Carlos III de Madrid



Coordenadas

**Hora** 11:00 - 12:00  
**Lugar** Seminario del Departamento de Matemáticas  
2.2 D08 Edificio Sabatini.

Dirección

Avda. de la Universidad 30  
28911, Leganés, Madrid

Department of Mathematics

